Adaptive Control with Integral Control
Action for Angular Position with Full State Measurable on the D.C. Motor

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Abstract

In this project, an adaptive controller with integral control action is designed for a D.C. motor control system, where the full state of the system, $x$, is measurable. The exact values of the transfer function coefficients are not known, however they are approximated through the calibration steps in the previous project. The control system will be verified by observing the behavior of the D.C. motor. Since the approximated transfer function coefficients are known, a simulation is done using MATLAB in order to compare to the real hardware system. Also, the effect of different choices of the adaptation gain matrix, $\Gamma$, is investigated and discussed. Lastly, the augmented system is tested with step disturbance.
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Chapter 1

Adaptive Controller Design

To implement adaptive control law with integral action, the actual plant parameters are not needed (this is an adaptive system, anyway). Previously calibrated values in CA3 project for $K$ and $\tau$ in the transfer function are for simulation purposes to verify the controller. The adaptive controller can be designed with the following steps:

1. Plant augmentation by incorporating a new state, $x_I$;
2. Design control law;
3. Choose a reference model for the system to track;
4. Applying matching conditions between the adaptive plant and the reference model;
5. Design adaptive law; and
6. Apply boundedness analysis.

1.1 Plant Augmentation

The suitable state-variable description for the D.C. motor is given by

\[
\begin{align*}
x_1 &= y \\
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= -\frac{1}{\tau}x_2 + \frac{K}{\tau}u
\end{align*}
\]

or in the state space representation,

\[
\dot{x}_p = A_p x_p + gbu \tag{1.1}
\]

where $x_p = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $A_p = \begin{bmatrix} 0 & 1 \\ 0 & -1/\tau \end{bmatrix}$, $b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, and $g = \frac{K}{\tau}$. Consider an additional state

\[
\dot{x}_I = y - r \tag{1.2}
\]
where $r(t)$ is the reference signal, we have the augmented plant

$$
\begin{bmatrix}
\dot{x}_p \\
\dot{x}_I
\end{bmatrix} = 
\begin{bmatrix}
A_p & 0 \\
1 & 0.0 & 0
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_I
\end{bmatrix} + g
\begin{bmatrix}
b \\
0
\end{bmatrix} u + 
\begin{bmatrix}
0 \\
-1
\end{bmatrix} r
$$

(1.3)

### 1.2 Control Law Design

Consider a non-adaptive state feedback control law,

$$u = \theta^* x_p = \theta_1^* x_1 + \theta_2^* x_2 + \theta_I^* x_I$$

(1.4)

The closed-loop system will then be

$$
\begin{bmatrix}
\dot{x}_p \\
\dot{x}_I
\end{bmatrix} = 
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_p \\
x_I \\
x_1 \\
x_2 \\
x_I
\end{bmatrix} + 
\begin{bmatrix}
0 \\
0 \\
0 \\
1 \\
-1
\end{bmatrix} r
$$

(1.5)

However, for an unknown system, the exact control gains $\theta^*$ are not possible to be defined. Here, we define a time varying control gains $\theta_1(t)$, $\theta_2(t)$, and $\theta_I(t)$ such that the control law,

$$u(t) = \theta_1(t)x_1(t) + \theta_2(t)x_2(t) + \theta_I(t)x_I(t)$$

(1.6)

has the same form as the non-adaptive feedback control.

### 1.3 Reference Model

A good reference model should be chosen to force the plant to behave as required. Since the augmented plant is a third order plant, a third order reference model with second order dominant pole placement is designed,

$$y_m = \bar{G}_m r$$

(1.7)

where

$$\bar{G}_m(s) = \frac{\lambda \omega_n^2}{(x + \lambda)(s^2 + 2\zeta \omega_n s + \omega_n^2)}$$

(1.8)

The speed of response is defined by natural frequency, $\omega_n$, and damping specified by damping coefficient, $\zeta$. Here $\lambda$ will be designed 10 times as large as the 2 poles for fast rate of convergence. We represent the reference model in the state-space

$$
\begin{bmatrix}
\dot{x}_m \\
y
\end{bmatrix} = 
\begin{bmatrix}
A_m & 0 \\
1 & 0.0 & 0
\end{bmatrix}
\begin{bmatrix}
x_m \\
x_p
\end{bmatrix} + g
\begin{bmatrix}
1 \\
0
\end{bmatrix} u + 
\begin{bmatrix}
0 \\
-1
\end{bmatrix} r
$$

(1.9)
Note that this particular state-space representation is chosen for the ease of matching condition step in the next section.

### 1.4 Matching Conditions

Consider the plant

\[
\dot{x}_p = \begin{bmatrix} 0 & 1 & 0 \\ \theta_1^* g & \theta_2^* g - \frac{1}{\tau} & \theta_I^* g \end{bmatrix} \bar{x}_p + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r
\]

\[
y = [1 \ 0 \ 0] \bar{x}_p
\]  

(1.10)

and the reference model

\[
\dot{x}_m = \begin{bmatrix} -(\omega_n^2 + 2\zeta \omega_n \lambda) & 1 \\ -\lambda + 2\zeta \omega_n & 0 \end{bmatrix} \bar{x}_m + \begin{bmatrix} 0 \\ 0 \end{bmatrix} r
\]

\[
y = [1 \ 0 \ 0] \bar{x}_p
\]  

(1.11)

Assuming that for the perfect gains, the feedback control achieves model matching when

\[
\begin{bmatrix} 0 & 1 & 0 \\ \theta_1^* g & \theta_2^* g - \frac{1}{\tau} & \theta_I^* g \end{bmatrix} = \begin{bmatrix} -(\omega_n^2 + 2\zeta \omega_n \lambda) & 1 \\ -\lambda + 2\zeta \omega_n & 0 \end{bmatrix}
\]

(1.12)

or

\[
\theta_1^* = -\frac{1}{g}(\omega_n^2 + 2\zeta \omega_n \lambda) \\
\theta_2^* = -\frac{1}{g}(\lambda + 2\zeta \omega_n - \frac{1}{\tau}) \\
\theta_I^* = -\frac{1}{g}(\lambda \omega_n^2)
\]  

(1.13) (1.14) (1.15)

### 1.5 Adaptive Law Design

An adaptive law is designed to control the time-varying gains as follows

\[
\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_I \end{bmatrix} = -sgn(g) \Gamma \begin{bmatrix} x_1 \\ x_2 \\ x_I \end{bmatrix} \tilde{e}^T \bar{P} \bar{b}
\]

(1.16)

where \( \Gamma \) is a \( 3 \times 3 \) adaptive gains matrix, \( \tilde{e} = \bar{x}_p - \bar{x}_m \) is the \( 3 \times 1 \) state error vector, and \( \bar{P} \) is the \( 3 \times 3 \) symmetric positive definite solution of the Lyapunov equation

\[
\bar{A}_m^T \bar{P} + \bar{P} \bar{A}_m = -\bar{Q}
\]

(1.17)

where \( \bar{Q} \) is any \( 3 \times 3 \) symmetric positive definite matrix. Here we see that the reference model \( \bar{A}_m \) must be chosen to be a Hurwitz matrix in order for the Lyapunov equation to hold true.
1.6 Boundedness Analysis

For the design of the control and adaptive law shown in the previous sections, here we must show that the control methodology works well for this system. In other words, the error signal dynamic must be bounded and converge to zero in steady-state.

Consider the state error,
\[ \bar{e} = \bar{x}_p - \bar{x}_m \]

We have the error signal dynamic
\[
\dot{\bar{e}} = \bar{A}_m \bar{e} + g \bar{b} \phi_x^T x_p + g \bar{b} \phi_I x_I \\
= \bar{A}_m \bar{e} + g \bar{b} \phi^T \bar{x}_p
\]
(1.18)

where \( \phi_x = \begin{bmatrix} \theta_1(t) - \theta_1^* \\ \theta_2(t) - \theta_2^* \end{bmatrix}, \phi_I = \theta_I(t) - \theta_I^* \), and \( \phi = \begin{bmatrix} \phi_x \\ \phi_I \end{bmatrix}, \bar{x}_p = \begin{bmatrix} x_p \\ x_I \end{bmatrix} \).

It is now a well-known error equation with \( \bar{b} = \begin{bmatrix} b \\ 0 \end{bmatrix} \) is known. We then consider the following Lyapunov function candidate
\[
V = \bar{e}^T \bar{P} \bar{e} + |g| \phi^T \Gamma^{-1} \phi
\]
(1.19)

By evaluating \( \dot{V} \) along the trajectory of the system, we have
\[
\dot{V} = -\bar{e}^T Q \bar{e}
\]
(1.20)

As we have defined \( \bar{Q} \) as a positive definite matrix in earlier section, we have \( \dot{V} \leq 0 \), and thus \( V(t) \) is bounded. It implies that \( \dot{e}, \bar{e}, \) and \( \phi \) are bounded, and thus \( \lim_{t \to \infty} \bar{e} = 0 \).
Chapter 2

Simulation Results

Before starting with the real controller on the D.C. motor, since we have already obtained the model of the motor, a simulation is done in order to obtain a good adaptive controller. In this simulation, a simple MATLAB and Simulink file is written and a square wave of amplitude ±1 is input to the system. The complete MATLAB and Simulink source codes are included in the last chapter of this report.

2.1 Plant Parameters

For the first simulation, the model plant is simulated as

$$G_m(s) = \frac{(25)(50)}{(s^2 + 9s + 25)(s + 50)}$$

(2.1)

corresponds to $\zeta = 0.9$ and $\omega_n = 5$. This model plant is chosen as it is highly damp and it has a fast settling time of 0.9s.

For the positive definite matrix $\bar{Q}$, it is chosen as

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(2.2)

and the solution of Riccati equation gives us

$$P = \begin{pmatrix} 5.6675 & 0.0245 & 11.1358 \\ 0.0245 & 0.0089 & 0.0004 \\ 11.1358 & 0.004 & 30.8106 \end{pmatrix}$$

(2.3)

As for the gain, $\Gamma$, it is chosen to be a diagonal matrix such that each value at the diagonal has its own physical meaning. Each value at the diagonal of $\Gamma$ is correspond to the weight putting on the respective signal ($x_1$, $x_2$, or $x_I$). The higher weight says that the signal is more significant than the other and more control effort will be put on it. In this simulation,

$$\Gamma = diag(1, 1, 1)$$

(2.4)
2.2 Results and Discussions

Simulation results are shown in Fig. 2.1 and Fig. 2.2. As we observe from the output plot together with the reference model output, we noticed that the controller does not manage to control the plant output to track the reference signal. Looking at the position and velocity error plots, we see that the errors are decreasing, and they seem to be converging to zero. As for the adapted gains shown in Fig. 2.2, since the exact gains can be calculated in the simulation, we have compared the simulated gains with the exact calculated gains. Here it seems that the simulated gains do not converge to the exact calculated gain as theoretically proven, thus, it is suspected that matrix $\tilde{Q}$ is too small for the system’s parameters to converge to its values in short time.

With different $\Gamma$

In order to improve the system’s response, the gain value, $\Gamma$, is replaced by a higher value, with 100 times the original $\Gamma$ used. Here,

$$\Gamma = diag(100, 100, 100) \quad (2.5)$$

Simulation result is shown in Fig. 2.3 and Fig. 2.4 respectively. The output plant managed to track the reference model at steady-state with no steady-state errors. As we know that $\Gamma$ determines the convergence rate of the adapted gains, the higher $\Gamma$ is, the faster the gains converged to the exact value (compare Fig. 2.4 with Fig. 2.2), and thus the faster the output plant meets the reference model. However, we see that higher control signal is needed in order to force the system to track the reference model.

Another approach to reduce the control effort of the system at the same time maintaining the system response, is to choose different adaptive rate for different signal. In this case, since the velocity of the motor is not a concern for us in the project, we assign the following values to $\Gamma$,

$$\Gamma = diag(50, 1, 100) \quad (2.6)$$

Here, the state $x_I$ is treated as the most important state as it is the augmented state which determine the error of the system, while $\gamma_{11}$ is our secondary interest as it directly determine the rate of convergence of the position output. Simulation result is shown in Fig. 2.5 and Fig. 2.6. According to the result, we see that the response of the system is of similar response to the previous one. However, for the control signal, it shows a lower control effort (from 5 to about 2) needed to control the system. From the velocity error plot, we see that the velocity error is larger in this case, but it does not affect the system output significantly. The oscillation occurs in the transient response of the system is due to the higher gains we have assigned to the system. Here we see that even with such high gain, the transient response of the system is not satisfying as it does not track the reference model well during the rising and falling of the signal.
With different $\tilde{Q}$ matrix

Theoretically, from the differential of Lyapunov function derived in the previous chapter, when matrix $\tilde{Q}$ is diagonal,

$$\dot{V} = -\tilde{e}^T \tilde{Q} \tilde{e} = -\tilde{q}_{11} \tilde{e}_1^2 - \tilde{q}_{22} \tilde{e}_2^2 - \tilde{q}_{33} \tilde{e}_3^2$$  \hspace{1cm} (2.7)

We see that the higher value of $q_{11}$, $q_{22}$ and $q_{33}$ would lead to a faster rate of convergence to the Lyapunov function, which is the sum of weighted squared errors. In other words, the higher value the matrix $\tilde{Q}$ is, the faster the state errors and adapted gains converged to zero and its exact values respectively. Here, we adopt the following $\tilde{Q}$,

$$\tilde{Q} = \text{diag}(50, 50, 50)$$  \hspace{1cm} (2.8)

Simulation result is shown in Fig. 2.7 and Fig. 2.8. As compared to the previous plots, we see that the response of the system is much better than the previous, as the errors of the system are of smaller order. The system output managed to track the reference model almost perfectly after just 1 cycle. For adapted gains wise, it is also shown that the gains now converges to the exact gains in a faster response. However, similar to the $\Gamma$ value, the higher value it is, the larger control effort needed to control the system. Although it might work well in simulation, it will cause problem to the real D.C. motor system as it will be shown in the next chapter.

Presence of step disturbances

In this subsection, the tuned controller above is tested with step disturbance at the input signal with frequency of 0.1 Hz. The block diagram of the step disturbance in the simulation program Simulink is added as shown below

![Block diagram](image)

The simulation result is shown in Fig. 2.9 and Fig. 2.10. We see that the controlled system manage to reject the step disturbance almost immediately when it occurs. It proved that the
adaptive controller has completely rejected the disturbance in a very short time and there is no steady-state error at all.
Figure 2.1: Simulation results with $\bar{Q} = \text{diag}(1,1,1)$ and $\Gamma = \text{diag}(1,1,1)$
Figure 2.2: Adapted gains with $\hat{Q} = \text{diag}(1, 1, 1)$ and $\Gamma = \text{diag}(1, 1, 1)$
Figure 2.3: Simulation results with $\bar{Q} = \text{diag}(1, 1, 1)$ and $\Gamma = \text{diag}(100, 100, 100)$
Figure 2.4: Adapted gains with $\bar{Q} = \text{diag}(1, 1, 1)$ and $\Gamma = \text{diag}(100, 100, 100)$
Figure 2.5: Simulation results with $\bar{Q} = \text{diag}(1, 1, 1)$ and $\Gamma = \text{diag}(50, 1, 100)$
Figure 2.6: Adapted gains with $\bar{Q} = diag(1, 1, 1)$ and $\Gamma = diag(50, 1, 100)$
Figure 2.7: Simulation results with $\tilde{Q} = \text{diag}(50, 50, 50)$ and $\Gamma = \text{diag}(50, 1, 100)$
Figure 2.8: Adapted gains with $\bar{Q} = \text{diag}(50, 50, 50)$ and $\Gamma = \text{diag}(50, 1, 100)$
Figure 2.9: Simulation results with $\bar{Q} = \text{diag}(50,50,50)$ and $\Gamma = \text{diag}(50,1,100)$ with disturbance
Figure 2.10: Adapted gains with $\bar{Q} = diag(50, 50, 50)$ and $\Gamma = diag(50, 1, 100)$ with disturbance
Chapter 3

Experimental Results

Before the start of the real hardware testing, the following modification on the Labview program is made:

1. MATLAB scripts to calculate matrix $\tilde{P}$ to be fed directly into the system parameters are added as shown below. The user can then alter the damping ratio and natural frequency of the reference model, the $\tilde{Q}$ matrix, and the $\Gamma$ matrix directly at the front panel.
2. Important data such as $x_1$, $e_1$, $u$, and $\theta$ are logged into a text file. The text file will then be imported into MATLAB environment to be plotted, as to compare with the simulation result. Also, it has the advantage of showing the whole transient response of the adapted system, as to show how fast it converge to the desired value, while the original Labview program only shows a portion of the response.
3. The control law, reference, and adaptive law block diagrams in the Labview environment are modified to include the augmented state $x_I$. 
Thus, in this chapter, the graphical user interface (GUI) or the front panel of this Labview program is not used, as the data will be imported to MATLAB environment to display and analyse.

### 3.1 Results and Discussions

As shown in the last chapter, the following values

\[
\bar{Q} = \text{diag}(50, 50, 50) \quad (3.1)
\]

\[
\Gamma = \text{diag}(50, 1, 100) \quad (3.2)
\]

will produce good control response in the simulation. Thus, for the first trial using the real D.C. motor hardware, the same values are adopted. The output response, together with position and velocity errors, control signals, and the adapted gains are plotted in Fig. 3.1 and Fig. 3.2.

As observed from the plots, we noticed that the output response does not behave exactly the same way as the simulation result shown in Fig. 2.7 and Fig. 2.8 from the previous chapter. Particularly, the control effort of the system is undesirably large (> 50). Based on the observation and some theoretical knowledge, the large control effort was caused by large gains of the system. Equivalently, larger gains will force the output signal to track the reference more heavily, and thus causing the large control effort especially at the rising and falling edge of the reference signal.

In order to obtain a better result, further fine tunings are needed. Both matrix \( \bar{Q} \) and matrix \( \Gamma \) need to be considered in the designed controller. However, in the simulation part done
in the previous chapter, we have already shown that $\Gamma = \text{diag}(50, 1, 100)$ able to produce a good control result in terms of theoretical and simulation. Thus, in next subsection, matrix $\hat{Q}$ is varied to see the effect on the controller.

**Variation of $\hat{Q}$**

In this section, we’ll investigate the effect of different $\hat{Q}$ on the controlled plant. Theoretically, $\hat{Q}$ is used to compute the value of $\hat{P}$ in the Lyapunov equation. It does but not directly affect the convergent rate of the position and velocity errors. In our first trial, the value $\hat{Q}$ is chosen as

$$\hat{Q} = \text{diag}(25, 25, 25) \quad (3.3)$$

while value $\Gamma$ remain unchanged from the last one.

Results are shown in Fig. 3.3 and Fig. 3.4. According to the plots, this set of $\hat{Q}$ decrease the control effort $u$ of the system. However, it is still not satisfying as it has exceeded the saturation value 5. To complete the test on $\hat{Q}$ value, another set of experiment is done with

$$\hat{Q} = \text{diag}(10, 10, 10) \quad (3.4)$$

The results are plotted in Fig. 3.5 and Fig. 3.6. The output plant manage to track the reference model rather well, with maximum position error maintaining at about $\pm 2$ during the transient response. Most significantly, the control signal now shows a much lower value as compared to the previous cases. To further improve the system, variation on $\Gamma$ value will be tested in the next subsection.

**Variation of $\Gamma$**

To further improve the system, $\Gamma$ value can be made bigger, to produce a faster error dynamic response. In this simulation, the $\Gamma$ value is increased to

$$\Gamma = \text{diag}(100, 1, 200) \quad (3.5)$$

Here, the gain for state $x_2$ remains unchanged as we only focusing on position control in this project. The result is plotted in Fig. 3.7 and Fig. 3.8. Based on the position error plot, it seem that the system output manage to track the reference model rather well, taking short time to reach the steady-state, with the maximum position error at about 0.2. However, looking at the control signal, again it has exceeded the maximum allowed value 5. Thus this set of $\Gamma$ value is not desired. The previous value will be retained in the next test.

**Presence of step disturbances**

In this case, we will do 2 specific tests on the adaptive controller. In these tests, a step disturbance with amplitude 1 and similar frequency with the step reference (with 90 degree phase shift) is applied to the input of the system. Here we tested the system with both
positive and negative step reference. The results are plotted in Fig. 3.9, Fig. 3.10, Fig. 3.11 and Fig. 3.12 respectively. Since the system is controlled in the presence of an integral controller, theoretically it will reject step disturbances. This is proven by the experimental results. We see that the disturbance has very minor impact to the output response, and the output response managed to achieve zero steady-state value even in the presence of disturbance. Here we conclude that the adaptive controller has completely rejected the disturbance in a very short time.
Figure 3.1: Output results with $\bar{Q} = diag(50, 50, 50)$ and $\Gamma = diag(50, 1, 100)$
Figure 3.2: Adapted gains with $\bar{Q} = \text{diag}(50,50,50)$ and $\Gamma = \text{diag}(50,1,100)$
Figure 3.3: Output results with $\bar{Q} = \text{diag}(25, 25, 25)$ and $\Gamma = \text{diag}(50, 1, 100)$
Figure 3.4: Adapted gains with $\bar{Q} = diag(25, 25, 25)$ and $\Gamma = diag(50, 1, 100)$
Figure 3.5: Output results with $\bar{Q} = \text{diag}(10, 10, 10)$ and $\Gamma = \text{diag}(50, 1, 100)$
Figure 3.6: Adapted gains with $\bar{Q} = \text{diag}(10, 10, 10)$ and $\Gamma = \text{diag}(50, 1, 100)$
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Figure 3.12: Adapted gains with $Q = \text{diag}(10, 10, 10)$ and $\Gamma = \text{diag}(100, 1, 200)$ and step disturbance 2
Chapter 4

Conclusion

In this project, I have re-designed a controller by augmented a integral action into the continuous-time D.C. motor position control system done in CA3. In this controller design, the exact values for the transfer function coefficients are not known, but are approximated through the calibration steps obtained in CA3 itself.

Firstly, the system is simulated in the MATLAB and Simulink environment using the following transfer function obtained through the calibration,

\[ H(s) = \frac{5.3340}{(0.3s + 1)(s)} \]

In the simulation, the effect of different choices of \( \Gamma \) and \( \bar{Q} \) matrix are investigated. It is shown that the value

\[
\begin{align*}
\Gamma &= \text{diag}(50, 1, 100) \\
\bar{Q} &= \text{diag}(50, 50, 50)
\end{align*}
\]

worked well in the simulation. Also, it is shown that by augmenting the system with an integral action, the augmented D.C. motor system has managed to track the reference model more closely.

Then, the controller is implemented to the real D.C. motor hardware available in the ECE laboratory. Again, the choices of \( \Gamma \) and \( \bar{Q} \) are investigated again with the real system. The experimental results shows that high value of \( \Gamma \) and \( \bar{Q} \) values will impose a high control effect which is not desired in the real system. The value of \( \Gamma \) and \( \bar{Q} \) are further fine tuned, and the final value of

\[
\begin{align*}
\Gamma &= \text{diag}(50, 1, 100) \\
\bar{Q} &= \text{diag}(10, 10, 10)
\end{align*}
\]

are found working well in the real hardware system. Lastly, we have also verified that the adaptive controller with integral action can reject step disturbances and ensure zero steady-state error.
Chapter 5

Source Codes

The following MATLAB and Simulink files are used in this project:

1. adaptive_sim.m
2. adaptive_real.m
3. simulation.mdl
4. part5.vi

adaptive_sim.m

This is a MATLAB script to simulate the adaptive controller in Chapter 2.

clear all;
z = 0.9;
w = 5;
h = 50;
Am = [0 1 0; -w^2-2*z*w*h -h-2*z*w -h*w^2;1 0 0];

Q11 = input('Q11 : ');
Q22 = input('Q22 : ');
Q33 = input('Q33 : ');
Q = [Q11 0 0;0 Q22 0; 0 0 Q33];
P = lyap(Am',Q)

G1 = input('Gamma11 : ');
G2 = input('Gamma22 : ');
G3 = input('Gamma33 : ');

% output
P12 = P(1,2)
P22 = P(2,2)
P32 = P(3,2)
%calculate real gains
theta1 = ((-w^2 - 2*z*w*h)/17.78)*ones(1,length(gains));
theta2 = ((-2*z*w+h+3.33)/17.78)*ones(1,length(gains));
thetai = (-h*w^2/17.78)*ones(1,length(gains));

t = 0:0.01:(length(gains)-1)/100;
figure(1)
subplot(4,1,1)
plot(t,states(:,4),t,states(:,1),'r--');
title('Reference model (red) and output plant (blue)');
subplot(4,1,2)
plot(t,states(:,4)-states(:,1));
title('position error');
subplot(4,1,3)
plot(t,states(:,5)-states(:,2));
title('velocity error');
subplot(4,1,4)
plot(t,u)
title('Control signal');
xlabel('Time (sec)');

figure(2)
subplot(3,1,1)
plot(t,theta1,'r--',t,gains(:,1));
legend('Exact gains','Simulated gains');
title('θ_1');
subplot(3,1,2)
plot(t,theta2,'r--',t,gains(:,2))
title('θ_2');
subplot(3,1,3)
plot(t,thetai,'r--',t,gains(:,3))
title('θ_I');
xlabel('Time (sec)');
This is a MATLAB script to plot the output for D.C. motor control system in Chapter 3.

clear all;
% load files from Labview spreadsheet
load e1.txt;
load e2.txt;
load ei.txt;
load t.txt;
load theta1.txt;
load theta2.txt;
load thetai.txt;
load u.txt;
load x1.txt;
load x2.txt;
load xi.txt;

% plot graphs
 t = (t - t(1))/1000;
figure(1)
 subplot(4,1,1); plot(t,x1);
title('Real output plant');
xlim([0 min(t(length(t)),120)]);
 subplot(4,1,2); plot(t,e1)
title('position error');
xlim([0 min(t(length(t)),120)]);
 subplot(4,1,3); plot(t,e2)
title('velocity error');
xlim([0 min(t(length(t)),120)]);
 subplot(4,1,4); plot(t,u)
title('Control signal');
xlabel('Time (sec)');
xlim([0 min(t(length(t)),120)]);

figure(2)
 subplot(3,1,1); plot(t,theta1);
title('$\theta_1$');
xlim([0 min(t(length(t)),120)]);
 subplot(3,1,2); plot(t,theta2)
title('$\theta_2$');
xlim([0 min(t(length(t)),120)]);
 subplot(3,1,3); plot(t,thetai)
title('$\theta_i$');
xlabel('Time (sec)');
xlim([0 min(t(length(t)),120)]);
simulation.mdl

This is a Simulink block diagram for the overall adaptive controller design for simulation in Chapter 2.
part5.vi

This is a Labview block diagram for calibration in Chapter 3.