Trajectory Planning for Improving Vision-Based Target Geolocation Performance Using a Quad-Rotor UAV

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Abstract—This paper describes a novel method to improve the target location accuracy through imaging it from an aircraft. This method focuses on improving estimation accuracy of heading angle bias to then improve geolocation performance. A Particle Swarm Optimization (PSO) algorithm is employed to derive an expression of optimal trajectory, which can be a guide for trajectory planning. Thanks to the maneuverability of quad-rotor unmanned aerial vehicles (UAVs), the aircraft is commanded to follow path generated by trajectory planning to acquire multiple bearing measurements of the ground object. The main result is that the aircraft’s heading angle bias can be more accurately estimated using trajectory planning. Hence, the target is more accurately geolocated. The efficacy of this technique is verified and demonstrated by simulation results and flight test.

Index Terms—Quad-rotor UAV, Trajectory planning, Vision-Based geolocation, PSO.

I. INTRODUCTION

Unmanned aerial vehicles (UAVs) are increasingly being used for a wide variety of missions, such as intelligence, surveillance and reconnaissance. Camera is one of most common sensors employed to acquire information about surroundings in order to achieve autonomy. Several research groups are using UAVs with vision systems, such as cargo transfer [1], object detection [2-4] and vision-aided navigation [5-7]. For the target localization application, target-localization work using a camera-equipped UAV was early reported in [8] and a real-time method to calculate the location of a fixed target detected by a gimbaled camera in a fixed-wing UAV equipped with a single-antenna GPS on board is described in [9]. Then a framework of vision processing for geolocation coupled with path planning to realize the coverage of the target is developed in [10]. Vision-based tracking geolocation system for long-endurance UAVs has been reported in [11] and geolocation of multiple targets from airborne video without terrain data is described in [12]. However, the aforementioned literatures don’t take into account sensor measurement bias. There are two solutions to handle this problem. One method of calculating the heading angle bias using visual measurements for a single UAV is developed based on the batch least squares technique [13]. The other method is used in extended information filter (EIF) and presented in decentralized manner to jointly estimate the sensor biases and the unknown point-of-interest location [14], where it indicated that not all states are observable using a single UAV, a stationary POI and a POI-centered fo the UAV. Hence, in this paper the batch least squares technique is employed to estimate the yaw bias by taking visual measurements of a stationary ground object. Based on the technique, we are focused on designing trajectories to improve the estimation performance of yaw bias and then enhance the target geolocation accuracy further.

Related research work devoted mainly on developing trajectory planning for the enhanced target estimation in [15-19]. An information potential is generated from Shannon information in [20] and used to develop a switched feedback control law for integrated sensor path planning and control. An optimized-visibility motion planning approach is proposed using an extended Kalman filter (EKF) and within this estimation framework a control law is derived to optimize the target tracking and robot localization performance [21]. An adaptive robotic control actions are provided based on maximizing the map information by simultaneously maximizing the expected Shannon information gain on the OG map and

NOMENCLATURE

$\beta$ = depression angle  
$XY_{fly}$ = flying area of aircraft  
$XY_{search}$ = search area of aircraft  
$(x_v, y_v, z_v)$ = Cartesian coordinates of aircraft  
$(\psi, \theta, \phi)$ = Euler angles of aircraft  
$\delta \psi$ = yaw-angle bias of aircraft  
$f$ = focal length of the camera  
$(\zeta, \xi)$ = line-of-sight angles  
$\alpha$ = radian of the orbit  
$\eta$ = spin angle of aircraft  
$s$ = level flight distance of aircraft  

Subscripts

$f$ = focal plane  
$b$ = body frame  
$p$ = ground object of interest  
$c$ = camera frame  

Superscripts

$n$ = navigation plane  
$b$ = body frame
minimizing the uncertainty in the SLAM process [22]. An information-theoretic approach to distributed and coordinated control of multi-robot sensor systems is described in [23], the control objective becomes maximization of these Fisher information gained by the system. The amount of the Fisher information is used as the performance metric and a receding horizon optimal control formulation is developed and solved for trajectories that yield maximum information [24]. These "Information based" trajectory planning approaches presented in these literatures [20-24] are based on the use of information as a measure of utility for taking control actions. These information measures are formally related to both Shannon information and Fisher information. These methods, however, are infeasible to trajectory optimization problems related to parameter selection. Some work has been done to optimally choose the trajectory parameters of the UAVs in target orbits and tracking problems by using information metrics [25]. This paper attempts to extend circular orbit optimization to more general trajectory configurations by taking account into a bounded FOV region and yaw bias estimation problems. Particular attention is given that the estimation performance of the parameter "yaw bias" is only considered in this paper, since estimation error covariance represents the correlation relationship among other parameters like the ground object position and we choose yaw bias estimation error to build the performance metric. This provides a simple straight-forward representation of parameter estimate performance based on these measurements which is being acquired along the trajectory. Trajectory planning is considered as generating an optimal trajectory, where the UAV flies to take the ground object’s bearing measurements over time to obtain best heading angle bias estimation. There is no doubt that trajectory planning can be viewed as optimization problem under certain index (i.e., minimizing the estimation error of the bias). Particle Swarm Optimization (PSO) is a population-based optimization algorithm and has been used in solving many optimization problems successfully [26-30]. Because of its relative fast convergence and global search property, PSO is used to obtain optimal choices of trajectory parameters.

In a continuation of our previous research [31,32], the contribution of this paper is that we design trajectory planning strategies to improve the target localization accuracy, considering FOV constraints and performance metric dependent on yaw bias estimation error and all the process steps including trajectory planning, data collection and bias estimation are online implemented in a UAV onboard NUC. More specifically, PSO is firstly computed offline to solve optimal trajectory and a general expression is listed as a guide for online trajectory planning. Then, trajectory planning is started onboard UAV once the ground object of interest (GOI) is detected. The main result is that the estimation accuracy of heading angle bias can be improved significantly by using the bearing measurements along the optimal trajectory. As a result, the target is more accurately geolocated than the one without trajectory planning. To clarify, the yaw measurement bias provided by the attitude-heading reference system (AHRS) and the bias may vary every time when the AHRS is initialized, and the assumption of unknown and constant bias is justified in the short time, as the case that UAV fly over a ground object.

The paper is organized as follows. In Section II, we give an overview of the geometry between a quad-rotor UAV and GOI, including yaw-angle bias estimation and field-of-view constraint. In Section III, PSO employed to solve optimal trajectory for different scenarios is described. In Section IV, we detail trajectory planning for the UAV according to different cases, followed by the results of simulations performed to validate the efficacy of the novel method. In Section V, the results obtained from the flight test data are presented to verify the method further. The paper ends with some concluding remarks.

II. THE GEOMETRY OF GEOLOCATION

The dynamic relationship between the UAV and the GOI will be discussed in two parts, which are respectively yaw-angle bias estimation and field-of-view constraint.

A. Yaw-angle bias estimation

The UAV target geolocation model can be visualized in Fig. 1. In mathematics, it can be expressed as

\[
[\begin{bmatrix} x_p \\ y_p \end{bmatrix}] = [\begin{bmatrix} x_v \\ y_v \end{bmatrix}] + \frac{(x_p-x_v) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} C_b \begin{bmatrix} x_f \\ y_f \\ f \end{bmatrix}}{(0,0,1) C_n^{\theta}} \tag{1}
\]

where \( x_f \) and \( y_f \) are the coordinates of the image of the GOI’s corresponding point \( p \) in the image plane. The navigation frame \( n \) has axes aligned with the directions of north (\( x \)), east (\( y \)) and down (\( z \)). \( (x_v, y_v, z_v)^T \) and \( (x_p, y_p, z_p)^T \) represent the UAV’s position and the GOI’s position in the navigation frame \( n \), respectively. \( f \) is the focal length of the camera. The UAV navigation state \( (x_v, y_v, z_v, \psi, \theta, \phi) \) is measured by the onboard GPS receiver and the AHRS. The rotation matrix \( C_b^{\theta} \) represents the transformation from the body frame \( b \) to the navigation frame \( n \).

For UAV level-flight mode, the pitch and roll angles of the UAV can be accurately measured using accelerometers, but the yaw-angle measurement will introduce large measurement...
errors when a miniaturized magnetic flux gate or a compass is employed. To facilitate the estimation of the critical yaw-angle bias $\delta \psi$, the pitch and roll measurement errors will be ignored in our studies and only the bias in the yaw angle is considered.

It is assumed that the GOI is identified and is traced from frame to frame using a feature-tracking algorithm [32]. Taking these multiple bearing measurements of the GOI ensures that the heading angle bias can be estimated using the optical method proposed in [13]. The process of the yaw-angle bias estimation is illustrated as follows.

We estimate the parameter $\gamma = [\gamma_1, \gamma_2]^T$ where

$$\gamma_1 = [x_p, y_p]^T$$

is the position of the ground object and

$$\gamma_2 = \delta \psi$$

is the bias in the yaw measurement provided by the AHRS. The measured variables are

$$y = [y_1, y_2]^T$$

where $y_1 = [x_v, y_v, z_v, x_f, y_f, \theta, \phi]^T$ and $y_2 = \psi$.

The measurement equation has the general form

$$\gamma_1 = F(y_1, y_2)$$

and the actual measurements are

$$z_1 = y_1 + w_1, w_1 \sim N(0, R_1)$$

$$z_2 = y_2 + \gamma_2 + w_2, w_2 \sim N(0, R_2)$$

where the Gaussian measurement noise covariances $R_1$ is the $7 \times 7$ symmetric positive-definite matrix and $R_2$ is a positive number. Inserting (3) and (4) into the measurement equation (2), we obtain the nonlinear measurement equation

$$\gamma_1 = F(z_1 - w_1, z_2 - (\gamma_2 + w_2))$$

Using Taylor’s theorem then yields the linearized measurement equation

$$F(z_1, z_2) \approx \gamma_1 + \frac{\partial F}{\partial y_2} |_{z_1, z_2} \cdot \gamma_2 + \frac{\partial F}{\partial y_1} |_{z_1, z_2} \cdot w_1 + \frac{\partial F}{\partial y_2} |_{z_1, z_2} \cdot w_2$$

Suppose that $N(\geq 2)$ bearing measurements of the GOI are taken at the discrete time $k=1, \ldots, N$, that is, $(z_1, z_2, \ldots, z_N)$, linear regression in the parameter $\gamma \in \mathbb{R}^3$ can be formulated as

$$\begin{bmatrix}
  F(z_1, z_2) \\
  \vdots \\
  F(z_N, z_N)
\end{bmatrix} = \begin{bmatrix}
  I_2, \frac{\partial F}{\partial y_2} |_{z_1, z_2} \\
  \vdots \\
  I_2, \frac{\partial F}{\partial y_2} |_{z_N, z_N}
\end{bmatrix} \gamma + W$$

where

$$W \sim N(0, R)$$

is the equation error with its covariance

$$R = \text{diag} \left\{ \left( \frac{\partial F}{\partial y_2} |_{z_k, z_k} \right)^T R_1 \left( \frac{\partial F}{\partial y_2} |_{z_k, z_k} \right), \left( \frac{\partial F}{\partial y_2} |_{z_k, z_k} \right)^T R_2 \left( \frac{\partial F}{\partial y_2} |_{z_k, z_k} \right) \right\}$$

From (7), the yaw bias can be solved effectively with the weighted least squares (WLS) method [33]. The partial derivatives are then obtained by

$$A_k = \frac{\partial F}{\partial y_1} |_{z_k, z_k}, B_k = \frac{\partial F}{\partial y_2} |_{z_k, z_k}$$

The parameter estimation is given by

$$\hat{\gamma} = \left[ \sum_{k=1}^{N} I_2 B_k^T \left( A_k R_1 A_k^T + B_k R_2 B_k^T \right)^{-1} [I_2, B_k] \right]^{-1}$$

and the parameter estimation error covariance is

$$P = \left[ \sum_{k=1}^{N} I_2 B_k^T \right] \left( A_k R_1 A_k^T + B_k R_2 B_k^T \right)^{-1} \left[ I_2, B_k \right]^{-1}$$

### B. Field-of-view constraint

A bounded FOV region hinges on depression angle $\beta$, field-of-view angle $FOV$ and UAV pose. Since $\beta$ and $FOV$ are certain, the UAV pose is only considered for this region. To maintain the same GOI in the camera field-of-view, the trajectory is limited in the space called Flying Area $XY_{fly}$: $\{ (x_v, y_v) | x_{inf} \leq x_v \leq x_{sup}, y_{inf} \leq y_v \leq y_{sup} \}$. Similarly, at an arbitrary point along the trajectory, potential GOI will be detected only if it appears in the coverage called Search Area $XY_{search}$: $\{ (x_f, y_f) | x_{inf} \leq x_f \leq x_{sup}, y_{inf} \leq y_f \leq y_{sup} \}$.

These areas for a forward-looking camera and a side-looking camera are different, so they will be discussed respectively. We define $(x_0, y_0)$ is the UAV current horizontal position in the body frame $b$ and $h$ is the relative altitude of the UAV above the GOI. Furthermore, we set current heading of the UAV to be $\psi = 0^\circ$. One obtains for forward-looking camera $XY_{search}$:

$$\{(x_1, y_1) | x_{1inf} \leq x_1 \leq x_{1sup}, y_{1inf} \leq y_1 \leq y_{1sup}\}$$

$XY_{fly}$:

$$\{(x_2, y_2) | x_{2inf} \leq x_2 \leq x_{2sup}, y_{2inf} \leq y_2 \leq y_{2sup}\}$$

One obtains for side-looking camera $XY_{search}$:

$$\{(x_1, y_1) | x_{1inf} \leq x_1 \leq x_{1sup}, y_{1inf} \leq y_1 \leq y_{1sup}\}$$

$XY_{fly}$:

$$\{(x_2, y_2) | x_{2inf} \leq x_2 \leq x_{2sup}, y_{2inf} \leq y_2 \leq y_{2sup}\}$$
where \((x_1, y_1)\) and \((x_2, y_2)\) represent the GOI and the UAV horizontal position restricted in Search Area and Flying Area in the body frame \(b\) respectively. The green triangle shows the trajectory in the Flying Area and the blue rectangle represents the Search Area at current point along the trajectory, as shown in Figs. 2 and 3.

### III. Optimal Trajectory

The ground object’s altitude is assumed to be known. Thus, the geolocation method described in Section II is transferred as 2-D case.

The main equation for the 2-D case is obtained in level-flight mode. The geolocation method described in Section II is transferred as 2-D case.

**TABLE I**

<table>
<thead>
<tr>
<th>Maneuver</th>
<th>Employed Camera</th>
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<tbody>
<tr>
<td>Overflight</td>
<td>Front</td>
</tr>
<tr>
<td>Flyby</td>
<td>Side</td>
</tr>
<tr>
<td>Loitering</td>
<td>Side</td>
</tr>
</tbody>
</table>

**P** represents the correlation relationship among three parameters \((x, y, \delta \psi)\) and we choose yaw bias estimation error \(e_{\delta \psi} = |\gamma - \gamma|\) to build the performance metric.

From the Appendix, \(A_k, B_k\) are related to the parameters \((\xi, \zeta, \psi)\). It is assumed that UAV flies in a constant altitude, we confine our attention to the horizontal plane \(xy\). It can be inferred that \(e_{\delta \psi}\) is only related to the parameters \((x_v, y_v, \psi)\) according to (11)-(13). So the performance metric is expressed by

\[
e_{\delta \psi} = J(x_v, y_v, \psi)
\]

In [25], the optimization problem is not generally analytically and must be done numerically under some specific setting. Similarly, in this paper PSO algorithm is used offline to numerically obtain optimal choice of trajectory parameters based on flight simulation data.

**A. Assumptions**

The following assumptions were made on the data collected: the UAV’s airspeed is \(V_a = 3.44\text{m/s}\), constant flight altitude above the GOI is \(h = 45\text{m}\), field-of-view angle is \(FOV = 30^\circ\), depression angle is \(\beta = 45^\circ\). GPS data acquisition frequency is \(f_{GPS} = 4\text{Hz}\), \(\sigma_\psi = \sigma_\delta = \sigma_c = 5^\circ\), the bias in the yaw-angle measurement is \(\Delta \psi = 30^\circ\), \(\sigma_c = 0.5^\circ\). the differential global positioning system (DGPS) horizontal uncertainty is \(\sigma_x = \sigma_y \simeq 0.6\text{m}\) and the baro altitude uncertainty is \(\sigma_z \simeq 1.5\text{m}\).

**B. Search the optimal path with PSO**

The PSO was first proposed by Kennedy and Eberhart in [34]. The algorithm simulates the behavior of birds foraging. Each particle has two characteristics of velocity and position. The process mainly includes four parts: particle initialization, evaluation process, state update and judgment process. The position and velocity vector of each particle is randomly initialized. The core of the evaluation process is to estimate the proximity of the particle’s current position to the desired optimal position. It is represented by a selected fitness function value, and the smaller the fitness value, the closer the particle is to the desired optimal position. The state update is divided into the velocity and the location update, and the global optimal particles are used as the guide and influence the update of each particle state, so that the particles gradually move towards the optimal position. In our implementation, the position of a particle represents a complete UAV trajectory. The flowchart of the PSO implementation is shown in Fig. 4.

The equations used to compute the velocity and position of a single particle at iteration are given below:

\[
v^t_{i+1} = w \cdot v^t_i + c_1 \cdot rand_1 \cdot (\alpha^t_{i,\text{GOI}} - \alpha^t_{i}) + c_2 \cdot rand_2 \cdot (\alpha^t_{\hat{g}} - \alpha^t_{i})
\]

\[
\alpha^t_{i+1} = \alpha^t_i + v^t_{i+1}
\]

where \(v^t_i\) is the velocity of the particle \(i\), and \(\alpha^t_{i}\) is its position at the iterative step \(t\). \(\alpha^t_{\text{GOI}}\) is the best position of the particle and \(\alpha^t_{\hat{g}}\) is the best position of the swarm at the iterative step \(t\). \(N_p\) is population size, \(N_c\) is maximum of the iterative steps \((1 \leq i \leq N_p, 1 \leq t \leq N_c)\), \(rand_1\) and \(rand_2\) are random values between 0 and 1; \(w\) is the inertia personal influence factor, \(c_1\) and \(c_2\) are the social influence factors. In our implementation, the parameters are set as follows:

\(N_p = 40, N_c = 100, w = 0.4, c_1 = c_2 = 2\).
The fitness function is obtained from (14)

$$f^i_t = J(x^i_t) = J(x_{v^t}, y_{v^t}, \psi_t^t)$$ (17)

Our implementation of the PSO is discussed in the following scenarios: Overflight, Flyby, and Loitering, as shown in Table I. For the sake of simplicity, the yaw-angle will be set to $\psi = 0$ for Overflight and Flyby cases and such that the fitness function is only related to $y_{v^t}$ parameter. Similarly, the radian angle will be set to $\alpha = 1.5\pi$, it is related to the radius $R$ for Loitering. Hence, our implementation of the PSO is simplified to solve single variable optimization problem, over the search space $\theta_f \in [\theta_f - \text{inf}, \theta_f - \text{sup}]$. The range of the single variable in these scenarios is listed in Table II. Also, Fig. 5 illustrates the generating of the particles initial population. The UA V follows the optimal trajectory to take multiple bearing measurements of the GOI. For each scenario, the relationship of the objective function and the iteration step of PSO is described in Fig. 6. The experiment results from simulation are provided and summarized in Table III.

For each scenario, there exits an optimal trajectory where the estimation of the yaw-angle bias can reach the optimal value. Compared with other scenarios, the optimal value of the estimation in loitering scenario is improved significantly. Optimal trajectory parameters are drawn which provide guidelines for simulation and real-time flight test. Specifically, trajectory planning is started to make UAV transit from current trajectory to optimal trajectory when a ground object is detected. It guarantees that the yaw bias estimation performance is improved by trajectory planning so that the accuracy of the target geolocation can be enhanced.

### IV. Trajectory Planning

The conclusions developed in Section III will be applied to UAV trajectory planning and to ensure that the accuracy of attitude-heading bias estimation can be effectively improved in the generated path. To make the algorithm of trajectory planning clear, the algorithm will be illustrated in the following steps:

**Step 1:** Calculate the current relative position of the GOI with respect to the UAV if the GOI is identified.

**Step 2:** Depending on onboard camera installation, it can be divided into two cases. For forward-looking camera, the UAV will fly a distance $s_1$ along the right side or $s_2$ along the left side to reach the starting point of the optimal trajectory. For side-looking camera, there are two options by the tradeoff between accuracy and computation cost. If flyby flight is chosen, the UAV flies a distance $s$ along the left side, else if loitering flight is chosen, the UAV firstly turns at an angle $\eta$ and then flies a distance $s_3$ along the right side or $s_4$ along the left side.

**Step 3:** The UAV follows the optimal trajectory to take multiple bearing measurements of the GOI.

**Step 4:** After the completion of the yaw bias estimation, the UAV

### TABLE II

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>Variable range</th>
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<tbody>
<tr>
<td>Overflight</td>
<td>$y_{f - \text{inf}} \leq y \leq y_{f - \text{sup}}$</td>
</tr>
<tr>
<td>Flyby</td>
<td>$y_{a - \text{inf}} \leq y \leq y_{a - \text{sup}}$</td>
</tr>
<tr>
<td>Loitering</td>
<td>$y_{a - \text{inf}} \leq R \leq y_{a - \text{sup}}$</td>
</tr>
</tbody>
</table>

### TABLE III

<table>
<thead>
<tr>
<th>Scenarios</th>
<th>$e_{x_{\psi^t}}/\text{deg}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Overflight($y = y_{y^t}$)</td>
<td>8.22</td>
</tr>
<tr>
<td>Flyby($y = y_{X_f - \text{inf}}$)</td>
<td>15.43</td>
</tr>
<tr>
<td>Loitering($R = R_{X_f - \text{sup}}$)</td>
<td>2.04</td>
</tr>
</tbody>
</table>

Fig. 4. The flowchart of the PSO

Fig. 5. The generating of the particles initial population

Fig. 6. The evolution curve of PSO
will fly forward to realize target localization.

Firstly, the parameters in Step 1 are listed as follows. Let \( p_b = [x_b, y_b, z_b]^T \) denote the relative position of the GOI with respect to the UAV in the body frame \( b \). Consider Fig. 1, the pinhole camera model with an assumption of fixed zoom and known camera intrinsic parameters is

\[
\begin{bmatrix}
x_f \\
y_f 
\end{bmatrix} = \frac{f}{z_b} \begin{bmatrix}
x_b \\
y_b 
\end{bmatrix} \quad (18)
\]

The relative altitude between the UAV and the GOI [35] is given by

\[ h = -z_b \sin \phi + x_b \sin \theta \cos \phi + y_b \cos \theta \cos \phi \quad (19) \]

where \( h \) and \( \theta, \phi \) are directly measured by onboard barometer and AHRS. Combining (18) with (19), the relative position \( p_b \) between UAV and GOI can be obtained.

Next, the parameters in Step 2 is analyzed under different conditions.

### A. forward-looking camera

The UAV flies a distance \( s_1 \) along the right side with the condition \( y_b \geq 0 \); otherwise, the UAV flies a distance \( s_2 \) along the left side, and then the UAV flies forward. The distances are defined as

\[ s_1 = s_2 = |y_b| \]

### B. side-looking camera

We define \( d_1 = y_b - y_{1s-\text{sup}} \) and \( d_2 = y_b - y_{1s-\text{inf}} \). If flyby flight is chosen, the UAV flies a distance \( s \) along the left side and then flies forward. One obtains

\[ s = |d_1| - \sigma_y \]

Else if loitering flight with radius \( R \) is chosen, the UAV firstly turns an angle \( \eta \) at the current point by utilizing easy rotation of the quad-rotor aircraft. One obtains

\[ \eta = \arctan \left( \frac{x_b}{y_b} \right) \]

where the clockwise rotation is taken as looking along the UAV axis \( z_b \) from the origin \( (\eta \geq 0) \), while a negative rotation acts in an opposite sense \( (\eta < 0) \). Then, the UAV flies a distance \( s_3 \) along the right side with the condition \( d_2 \geq 0 \), otherwise, it flies a distance \( s_4 \) along the left side. Then, the UAV orbits the GOI in the loitering trajectory with the radian \( \alpha \). Eventually, the UAV switches to fly forward for localizing a target. The parameters are defined as

\[
R = \sqrt{x_b^2 + y_b^2 + s_3}, (d_2 \geq 0) \\
R = \sqrt{x_b^2 + y_b^2 + s_4}, (d_2 < 0) \\
s_3 = |d_2| - \sigma_y, s_4 = |d_2| + \sigma_y
\]

Monte Carlo simulations were performed to validate the efficacy of the technique presented, with respect to the scenarios introduced in the preceding section. Typical runs are presented in Figs. 7 and 8, in which the solid circle shows the actual position of a GOI measured to estimate the bias, the black star is the target’s actual position, and the red asterisk represents the result of the geolocation of the target’s position. In Tables IV and V, the statistical results are reported in terms of the RMS error associated with the yaw-angle bias estimation error \( e_{\delta \psi} \) and resulting localization error \( e_{\delta y} \). From the simulation results, we can see that the target geolocation accuracy is enhanced using a one-shot measurement, provided that the yaw-angle bias is more accurately estimated based on the UAV trajectory planning.

V. FLIGHT TEST

The quad-rotor UAV used in this flight test is an experimental one called T-Lion designed by the Control Science Group of Temasek Laboratories at the National University of Singapore as shown in Fig. 9. It was specially cater to carry high payload (>2 kg) with a long-endurance flight (>20 mins). The T-Lion has the ability to fly on predefined waypoint and circular path autonomously, so it can be used as a test platform to validate our method. The T-Lion is equipped with power distributor, flight controller, onboard computer and gimbal controlled camera, as shown in Fig. 9. An Intel NUC mini-computer with a powerful i7 processor is selected as onboard computer and mounted in the T-Lion, and it is run in ROS system [36], where it enables image processing and trajectory planning in real-time during flight. A monocular camera is used for visual measurements during flight. The camera is set to point directly downwards at the GOI.

The proposed method is now verified through actual flight data. The flight scenario is shown in Fig. 10. The quad-rotor UAV is flying different trajectories in outdoor environment. For each trajectory, the UAV firstly rely on feature detection to take multiple measurements of GOI to obtain the estimation of the yaw bias and then localize the target accurately when it is pop up in the video. In order to identify the GOI and the target easily, one AprilTag [37,38] is used as the GOI and another AprilTag is mounted on the target. It should be noted that not all information provided by AprilTag are employed for this experiment but only the pixel position of the center point of AprilTag.

The UAV is commanded to fly all three trajectories presented in Section IV, as shown in Fig. 11. Specifically, there are common initial segments for the three trajectories. For the no planning case, the UAV is flying straight without changing direction of motion. For

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**TABLE IV**

<table>
<thead>
<tr>
<th>Estimation Performance for Forward-looking Camera</th>
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<tbody>
<tr>
<td>Overflight</td>
</tr>
<tr>
<td>No planning</td>
</tr>
<tr>
<td>Overflight planning</td>
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</table>

**TABLE V**

<table>
<thead>
<tr>
<th>Estimation Performance for Side-looking Camera</th>
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<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>No planning</td>
</tr>
<tr>
<td>Flyby planning</td>
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<tr>
<td>Loitering planning</td>
</tr>
</tbody>
</table>

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the flyby planning case, the UAV is flying along the right side to finish the transition from current trajectory to the optimal trajectory generated by flyby planning, when the GOI is detected. For the loitering planning case, the UAV is firstly flying along the left side to transit the optimal trajectory by loitering planning. Then, the UAV is changing to fly straight when the UAV obit of the GOI for one circle. The red trajectory represents the DGPS measurement of the UAV to provide the ground truth for the experiment. The arrows represent the yaw measured by the AHRS. The red square and red star represent the actual position of the GOI and the target measured by the DGPS respectively. The triangle is the UAV DGPS position, where the target is detected and geolocated. The black asterisk represents the result of target geolocation.

The following assumptions in the measurement process are used: $\sigma_x = \sigma_y = 0.2m$, $\sigma_z = 1m$, $\sigma_\theta = \sigma_\phi = \sigma_\psi = 1^\circ$ and $\sigma_\xi = \sigma_\zeta = 0.5^\circ$.

The assumption of the constant bias in the yaw measurement is justified according to Fig. 11. Before eliminating the bias, the arrows show significant angular offset with respect to the direction of motion. After having an estimate of the yaw angle bias, the angular offset decreases in varying extent for different trajectories. The main result is that the bias in the heading measurement is more accurately estimated using trajectory planning than no planning at all. As a result, the accuracy of target geolocation was enhanced greatly.

For the three cases shown in Fig. 11, $T_1$ represents the length of the video record selected with $N_1$ frames/data points available, in which the GOI is tracked frame by frame. The estimate of the bias in the yaw measurement and the localization errors of the target in the horizontal direction is reported in Table VI.

The successful results in actual flight trials has illustrated that the method proposed in this paper is capable of improving the yaw angle bias accuracy and obtaining a significant advantage in terms of target geolocation accuracy after eliminating the bias. Besides, the flight data results has proved the method is robust to the unknown sensor noise. Considering the computation cost, PSO is computed only once off line to get an optimal path, which can be a guideline for trajectory planning run onboard UAV. Hence, the method is practical for UAV real-time applications.
Fig. 10. Flight test: an outdoor scenario

Fig. 11. Target geolocation
VI. CONCLUSION

The geolocation accuracy of a target obtained from one-shot measurement is generally poor, due to the bias in the heading measurement provided by the AHRS onboard UAVs. In order to have a more accurate estimate of the heading angle bias, the proposed vision-based geolocation method relies on the UAV trajectory planning strategies to take multiple bearing measurements of a GOI along the generated trajectory. Finally, it can significantly improve the target geolocation accuracy.

From flight test results, it is shown that bearing measurements over time of the GOI taken in the generated trajectory enable more accurate calculation of heading angle bias and thus further enhanced target geolocation accuracy. With DGPS horizontal position accuracy 0.2 m, the position of the ground object and the target is more accurately geolocated with an increase of nearly 0.5 m. Compared with no planning, the bias in the heading measurement is generally poor, due to the bias in the attitude-heading measurement provided by the AHRS.

Appendix A

The aim of this Appendix is to provide the computation of the matrix $A_k$, $B_k$ presented in Section III. The main equation is

$$
\left[\begin{array}{c}
x_p \\
y_p
\end{array}\right] = \left[\begin{array}{c}
x_v \\
y_v
\end{array}\right] + \frac{(z_p - z_v)}{(0, 0, 1)C_v B_v} \left[\begin{array}{c}
1 \\
0
\end{array}\right] C_v B_v
$$

where

$$
C_v B_v = \left[\begin{array}{c}
\cos(\xi) \cos(\psi) \\
\cos(\xi) \sin(\psi) \\
\sin(\xi)
\end{array}\right]
$$

The parameters are estimated:

$$\gamma_1 = [x_p, y_p]^T$$

is the position of the ground object and

$$\gamma_2 = \delta\psi$$

is the bias error in the attitude-heading measurement provided by the AHRS.

The measured variables are

$$y = [y_1, y_2]^T$$

where $y_1 = [x_v, y_v, z_v, x_f, y_f, \theta, \phi]^T$ and $y_2 = \psi$.

The partial derivatives are obtained

$$A_k = \frac{\partial F}{\partial y_1} |_{z_k} = \left[\begin{array}{c}
1 \\
0
\end{array}\right] a_{13} \quad a_{14} \quad a_{15} \quad a_{16} \quad a_{17}$$

$$B_k = \frac{\partial F}{\partial y_2} |_{z_k} = \left[\begin{array}{c}
b_{11} \\
b_{21}
\end{array}\right]
$$

By setting $\theta = \phi = 0$, we obtain $A_k$, $B_k$ for the 2-D case as follows:

$$a_{13} = \cos(\psi + \xi) \cos \zeta \sin \zeta$$

$$a_{14} = (z_p - z) \frac{-\cos(\psi + \xi)}{\sin^2 \zeta}$$

$$a_{15} = (z_p - z) \frac{-\sin(\psi + \xi) \cos \zeta \sin \zeta}{\sin^2 \zeta}$$

$$a_{16} = (z_p - z) \frac{\cos \psi \sin^2 \zeta + \cos(\psi + \xi) \cos^2 \zeta \cos \xi}{\sin^2 \zeta}$$

$$a_{17} = (z_p - z) \frac{-\sin \psi \sin^2 \zeta - \cos(\psi + \xi) \cos^2 \zeta \sin \xi}{\sin^2 \zeta}$$

$$a_{23} = -\frac{\sin(\psi + \xi)}{\sin \zeta}$$

$$a_{24} = (z_p - z) \frac{-\sin(\psi + \xi)}{\sin^2 \zeta}$$

$$a_{25} = (z_p - z) \frac{\cos(\psi + \xi) \cos \zeta \sin \zeta}{\sin^2 \zeta}$$

$$a_{26} = (z_p - z) \frac{\sin \psi \sin^2 \zeta + \cos(\psi + \xi) \cos^2 \zeta \cos \xi}{\sin^2 \zeta}$$

$$a_{27} = (z_p - z) \frac{-\cos \psi \sin^2 \zeta - \cos(\psi + \xi) \cos^2 \zeta \sin \xi}{\sin^2 \zeta}$$

$$b_{11} = (z_p - z) \frac{-\sin(\psi + \xi) \cos \zeta \sin \zeta}{\sin \zeta}$$

$$b_{21} = (z_p - z) \frac{\cos(\psi + \xi) \cos \zeta}{\sin \zeta}$$

TABLE VI

<table>
<thead>
<tr>
<th>Target Geolocation Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1/s</td>
</tr>
<tr>
<td>-----------------------------</td>
</tr>
<tr>
<td>No planning</td>
</tr>
<tr>
<td>Flyby planning</td>
</tr>
<tr>
<td>Loitering planning</td>
</tr>
</tbody>
</table>

REFERENCES


